

Largeness and partial conservation

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Generalizing the celebrated Paris-Harrington theorem, Ketonen and Solovay [1] introduced α -largeness as a transfinite generalization of the notion of cardinality for finite sets. In particular, a finite set $F \subseteq \mathbb{N}$ is n -large if $\text{card } F \geq n$, and ω -large if $\text{card } F \geq 1 + \min F$.

This notion turns out to be an essential tool for proving conservation theorems for $\forall\Pi_3^0$ -sentences, where a sentence is $\forall\Pi$ if it is of the form $\forall X \phi(X)$ for some Γ -formula ϕ . Beyond partial conservation theorems, explicit α -largeness bounds computations for combinatorial theorems are of independent interest, as providing a finitary quantitative counterpart to countably infinite combinatorics.

In this talk, we will survey basic combinatorics of α -largeness together with its applications to $\forall\Pi_3^0$ -conservation theorems, and present a parameterized extension for $\forall\Pi_4^0$ -sentences. This is a joint work with Quentin Le Hou  rou and Keita Yokoyama.

[1] Ketonen, J., Solovay, R., *Rapidly growing Ramsey functions*, Ann. of Math. (2) 113(2), 267–314 (1981), <https://doi.org/10.2307/2006985>