

An extension of HA by sentential PEM

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Intermediate arithmetic is an arithmetic in between Heyting Arithmetic HA and Peano Arithmetic PA. Intermediate arithmetics are well-studied in the literature [1]. For example, [1] shows that all intermediate arithmetics over propositional logics with finite model property, has the so called de Jongh property.

An intermediate arithmetic IA, usually is constructed on the basis of some propositional axiom scheme $\phi(p_1, \dots, p_n)$ in the following way. IA is HA plus the universal closure of $\phi(A_1, \dots, A_n)$ for A_i being arbitrary formulas. Also recently, a hierarchy of such intermediate arithmetics have been considered by restriction of the substituted formulas to some Σ_n^0 or Π_n^0 -formulas [4, 3, 2]. However, an important variant is missing in such hierarchies: the sentential fragment of such intermediate arithmetics. In this study, in particular, we consider the extension of Heyting arithmetic by all instances of the Principle of Excluded Middle PEM by arithmetical *sentences*, called HA+PEM^s. We investigate the basic properties of HA+PEM^s. Among other things, we show that such intermediate arithmetic, has the same provability logic of PA.

[1] de Jongh, D., Verbrugge, R., and Visser, A. (2011), *Intermediate logics and the de Jongh property*, Archive for Mathematical Logic, 50:197–213.

[2] Fujiwara, M. (2026), *On the hierarchy of linearity axioms*, Archive for Mathematical Logic, 65(3):319–331.

[3] Fujiwara, M., Ishihara, H., Nemoto, T., Suzuki, N.-Y., and Yokoyama, K. (2023), *Extended frames and separations of logical principles*, Bulletin of Symbolic Logic, 29(3):311–353.

[4] Fujiwara, M. and Kurahashi, T. (2022), *Refining the arithmetical hierarchy of classical principles*, Mathematical Logic Quarterly, 68(3):318–345.