

Finitely non-standard models of Robinson arithmetic

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To the best of my knowledge, there is no systematic study of *finitely non-standard* models of *Robinson arithmetic*¹—that is, there is no systematic study of those models of Robinson arithmetic that have a non-empty finite set of *non-standard numbers*. I have made a modest attempt at such a systematic study (Lundstedt, [2]).

The main results are that for each such finitely non-standard model:

- The successor function is a permutation of the set of non-standard numbers (this follows trivially from the axioms). As is well-known, each permutation of a finite set has a unique decomposition into *cycles* on disjoint *orbits*.
- The restriction of addition to

$$\{\langle a, n \rangle : a \text{ non-standard, } n \text{ standard}\}$$

is “tame”, in the sense that this restriction is determined already by the permutation given by the successor function:

$$a + n = \text{the result of starting at } a \text{ and taking } n \text{ steps in its cycle.}$$

- The restriction of addition to

$$\{\langle \alpha, a \rangle : \alpha \text{ standard or non-standard, } a \text{ non-standard}\}$$

is “wild”. Contrary to the previous restriction, this one is not determined by the successor function, but subject to some constraints—in particular, its range may only consist of non-standard numbers.

- Similar to the case of addition, the restriction of multiplication to

$$\{\langle a, n \rangle : a \text{ non-standard, } n \text{ standard}\}$$

is determined by the successor function and addition, whereas the restriction of multiplication to

$$\{\langle \alpha, a \rangle : \alpha \text{ standard or non-standard, } a \text{ non-standard}\}$$

¹ I work with Robinson’s [1] original axiomatization:

$$\begin{aligned} &Sx \neq 0 \\ &Sx = Sy \rightarrow x = y \\ &x = 0 \vee \exists y : x = Sy \\ &x + 0 = x \\ &x + Sy = S(x + y) \\ &x \times 0 = 0 \\ &x \times Sy = x \times y + x. \end{aligned}$$

is not uniquely determined (by the successor function and addition), but subject to some constraints.

[1] Robinson, Raphael M. (1950), *An essentially undecidable axiom system*, Proceedings of the International Congress of Mathematicians, volume I; pages 729–730; American Mathematical Society; Providence 1952.

[2] Lundstedt, Anders E.V. (2025), *Finitely non-standard models of Robinson arithmetic*; version 2025-06-10. In: Natural induction (a model-theoretic study); PhD thesis in preparation.

URL:<https://anderslundstedt.com/research/#induction-drafts>