

# On the finite cohesiveness principle

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The *cohesiveness principle* (COH) plays an important role in the study of the reverse mathematics of Ramsey's Theorem for Pairs. It claims that, given a sequence of sets  $(R_i)_{i \in \mathbb{N}}$ , there exists an infinite cohesive set  $H$ , which means that it cannot be divided into two infinite subsets by elements of that sequence.

In this talk, we focus on instances of COH where the length of the given sequence is finite, referred to as fin-COH.

**Finite cohesive principle (fin-COH)** Let  $l \in \mathbb{N}$  and  $R = (R_k)_{k < l}$  be a sequence of sets of length  $l$ . Then there is an infinite set  $H$  such that for any  $k < l$ , we have either  $H \subseteq^* R_k$  or  $H \subseteq^* R_k^c$ . Here  $A \subseteq^* B$  abbreviates  $\exists b \forall x > b (x \in A \rightarrow x \in B)$ .

At first glance, such instances are trivial from a recursion-theoretic point of view. However, in the absence of enough induction or collection, the existence of cohesive sets for such instances becomes non-trivial. We show that over the weaker base theory  $\text{RCA}_0^*$ , fin-COH implies  $\Sigma_1^0$ , which answers a question by Fiori-Carones, Kołodziejczyk and Kowalik [2] and also by Belanger [1]. We also discuss the strength of fin-COH over  $\text{RCA}_0$ . In particular, we show that fin-COH is not provable over  $\text{WKL}_0$ .

## References

- [1] D. R. Belanger. Conservation theorems for the cohesiveness principle, 2022. Preprint.
- [2] M. Fiori-Carones, L. A. Kołodziejczyk, and K. W. Kowalik. Weaker cousins of Ramsey's theorem over a weak base theory. *Ann. Pure Appl. Logic*, 172(10):Paper No. 103028, 22, 2021.

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