Ramsey-like theorems for separable permutations

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Abstract

Following the discovery that the infinite version of Ramsey's theorem for pairs and two colors (RT^2_2) escaped a structural phenomenon from reverse mathematics known as the "Big Five", many other Ramsey-like statements have been studied from a computational and reverse mathematical viewpoint.

In this talk, we will be considering weakenings of RT^2_2 where, starting from an infinite graph, we search for an infinite subgraph that avoids some fixed pattern (a pattern will be a finite graph). Out of all the possible patterns, we will focus on those encoding permutations. In particular, our work will show the important role played by a certain class of permutation (already known in combinatorics for its many different characterizations) called the separable permutations.

Denoting by $\mathsf{RT}^2_2(p)$ the statement asserting that the pattern p can be avoided, our main result is that, in omega-models, $\mathsf{RT}^2_2(p)$ implies RT^2_2 if and only if p is a separable permutation.

This is joint work with Ludovic Patey.