

# TIGHTNESS AND SEMANTIC TIGHTNESS IN SUBTHEORIES OF PEANO ARITHMETIC

PIOTR GRUZA

*University of Warsaw, Institute of Mathematics*

The talk will be devoted to showing that certain subtheories of Peano Arithmetic possess properties that can be seen as forms of internal completeness or categoricity, known as tightness and semantic tightness.

We say that interpretations  $\sigma_1, \sigma_2: S \triangleleft T$  are isomorphic if there is a formula that acts in  $T$  as an isomorphism between the structures given by  $\sigma_1$  and  $\sigma_2$ . Theories  $S$  and  $T$  are bi-interpretable via  $\sigma: S \triangleleft T$  and  $\tau: T \triangleleft S$  if  $\tau \circ \sigma \simeq \text{id}_T$  and  $\sigma \circ \tau \simeq \text{id}_S$ . A theory  $T$  is tight if any two bi-interpretable  $\mathcal{L}_T$ -theories containing  $T$  are equal. Tightness can be thought of as a form of internal or local completeness of a theory: a tight theory has no two distinct extensions that can ‘see each other’, in the sense of bi-interpretability. Semantic tightness is a semantic analog of tightness; it is defined in terms of interpretations between models and can be thought of as a form of internal categoricity. One can show that if a theory is semantically tight, then it is tight.

Partly building on the work of Visser [1], Enayat proved that PA,  $Z_2$ , ZF, and KM have all of the above properties [2]. On the other hand, it was shown later by many authors that a number of natural subtheories of the theories mentioned above – including all of their finitely axiomatizable fragments – do not have any of these properties [2, 3, 4, 5]. These authors posed variants of a natural question: ‘Is there a proper subtheory of PA,  $Z_2$ , ZF, or KM that has any of these properties?’. During the talk, I will answer this question in the case of PA by showing that for every natural number  $n$  there is a semantically tight, computable subtheory of PA containing  $I\Sigma_n$  but not  $B\Sigma_{n+1}$ . I will also give an example of a tight but not semantically tight subtheory of PA and discuss possible strengthenings of the main result.

The talk will be based on joint work with L. Kołodziejczyk and M. Łełyk.

## REFERENCES

- [1] Albert Visser. CATEGORIES OF THEORIES AND INTERPRETATIONS. In Ali Enayat et al. (eds.), "Logic in Tehran", Cambridge University Press 2006, pp. 284–341.
- [2] Ali Enayat. VARIATIONS ON A VISSERIAN THEME. In Jan van Eijck et al. (eds.), "Liber amicorum Alberti: A tribute to Albert Visser", College Publications 2016, pp. 99–110.
- [3] Alfredo Freire, Joel Hamkins. BI-INTERPRETATION IN WEAK SET THEORIES. The Journal of Symbolic Logic 2021, vol. 86, pp. 609–634.
- [4] Alfredo Freire, Kameryn Williams. NON-TIGHTNESS IN CLASS THEORY AND SECOND-ORDER ARITHMETIC. The Journal of Symbolic Logic 2023, First View, pp. 1–28.
- [5] Ali Enayat, Mateusz Łełyk. CATEGORICITY-LIKE PROPERTIES IN THE FIRST ORDER REALM. Journal for the Philosophy of Mathematics 2024, vol. 1, 63–98.
- [6] Piotr Gruza, Leszek Kołodziejczyk, Mateusz Łełyk. TIGHTNESS AND SOLIDITY IN FRAGMENTS OF PEANO ARITHMETIC. In preparation.