Decision Problems for Recognizable Languages of Infinite Pictures

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Languages of infinite words accepted by finite automata were first studied by Büchi to prove the decidability of the monadic second order theory of one successor over the integers. Since then regular ω-languages have been much studied and many applications have been found for specification and verification of non-terminating systems, see [9, 8] for many results and references.

Altenbernd, Thomas and Wöhrle have considered in [1] acceptance of languages of infinite two-dimensional words (infinite pictures) by finite tiling systems, with the usual acceptance conditions, such as the Büchi and Muller ones, firstly used for infinite words. This way they extended both the classical theory of ω-regular languages and the classical theory of recognizable languages of finite pictures, [6], to the case of infinite pictures.

Many classical decision problems are studied in formal language theory and in automata theory and arise now naturally about recognizable languages of infinite pictures.

Using results of Castro and Cucker on decision problems for ω-languages of non deterministic Turing machines [3], and some topological arguments, we give the exact degree of numerous undecidable problems for Büchi-recognizable languages of infinite pictures. In particular, the non-emptiness and the infiniteness problems are Σ_1^1-complete, and the universality problem, the inclusion problem, the equivalence problem, the complementability problem, the determinizability problem and the unambiguity problem are all Π_2^1-complete. These decision problems are then located at the first or at the second level of the analytical hierarchy, and "highly undecidable".

This gives new natural examples of decision problems located at the first or at the second level of the analytical hierarchy.

We show also that topological and arithmetical properties of Büchi-recognizable languages of infinite pictures are highly undecidable.

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